There are no Limits of Reason

by

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Abstract

In his article "The Limits of Reason", G. J. Chaitin argues that there are limits of reason and that there can never be a theory of everything for all of mathematics. I claim that his argument involves either an equivocation, where the word *reason* is used with two different meanings, or a false hidden premise that does not account for all modes of reason. I use Peirce's modes of reasoning in support of my argument. I also argue that the difficulty of arriving at a theory of everything for all of mathematics, or even physics, is not due to any limits of reason, because there is none, but due to a plurality of abducted facts or hypotheses.

1. Introduction

In his article "The Limits of Reason", C. J. Chaitin argues that there are mathematical facts that are true for no reason and that the only way to use these facts is to assume them as axioms without using reasoning at all [1]. Actually, there are an infinite number of such mathematical facts that, as he claims, are logically irreducible. Chaitin states:

> "…certain mathematical facts are true for no reason, a discovery that flies in the face of the principle of sufficient reason".

The above statement makes sense when *reason* is taken to mean *cause, proof or explanation*. In this sense, the principle of sufficient reason is violated due to the existence of these facts. For example, experiments in Quantum Mechanics have shown that elementary particles have spontaneous behavior that violates the principle of causality. These are facts that violated the principle of sufficient reason.

However, Chaitin takes his argument one step further in an attempt to show that there are limits of reason and hence there can never be a theory of everything. He writes:

> "Using reason has certainly been an extremely fruitful approach, leading to modern mathematics and mathematical physics and all that goes with them…So I am saying that this approach that science and mathematics has been following for more than two millennia crashes and burns? Yes, in a sense I am. My counterexample illustrating the limited power of logic and reason, my source of an infinite stream of unprovable mathematical facts, is the number that I call omega."

Let us next attempt to reproduce Chaitin's argument as follows:

(1) – There exist mathematical facts that are true for no reason
(2) - The principle of sufficient reason is violated (from 1)
(3) - There are limits of reason (from 1 and 2)
(4) - Science and Mathematics use reason
(5) - Science and Mathematics are limited (from 3 and 4)

If *reason* in (1) and (2) means cause, proof or explanation but in (3) it means a type or process of thought that is much more than what is included in (1) and (2), then an equivocation is present. For example, consider the following question: Do women have to worry about man-eating sharks? This is a well-known equivocation because the word *man* is used with two different meanings, (a) a member of the human race or (b) a male member of the human race.

Now, let us consider this question: is reason limited because we are cannot find a reason for the truth of certain mathematical facts?

The above is an equivocation if the word *reason* is used with two different meanings in this order; the first is *reason as a type or process of thought* and the second is *reason as proof, explanation or cause*.

Now, the equivocation can be avoided if reason, as a type or process of thought, is identical to causes, proofs, and explanations in a logical sense. I believe Chaitin makes this assumption in his argument, in the form of a hidden premise for his conclusion (3) to be true. I will show in section 2, however, that this hidden premise is false because the facts that are true for no reason are results of reason. Then, reason, as a type or process of thought, is not identical to causes, proofs, and explanations in a logical sense but to facts, causes, proofs, and explanations. Chatin's argument can be declared unsound in this case. However, his proposition that "there can never be a theory of everything for all of mathematics" may still be true.

2. Modes of Reason

Two basic modes of reason are *deduction* and *induction*. In simple terms, a deduction is an inference that generates conclusions from general rules and facts. An inductive inference uses facts and conclusions to arrive at (probable) general rules. The Ancient Greeks and especially Aristotle and Euclid placed high emphasis on deductive inferences. Their respective collective works, *Organon* and *The Elements*, paved the way to modern formal systems used in mathematics and physics for theorem proving and for deriving predictions from a set of rules, called principles or axioms, and a set of propositions, called facts or premises. However, another powerful mode of reason is inductive inference and Isaac Newton was probably the first who stated elegantly its importance in physical science [2]:

> In this [experimental] philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation, were discovered.

Deduction and induction are not the only modes of reason. There is another very important mode detailed by Peirce and it is called abduction, or hypothesis [3]. Below are examples of how the three modes of reason work, in principle.

In the following example of deduction, a rule and a fact are used to arrive at a conclusion. The fact is the outcome of an experiment, in this particular case the experiment of watching who comes out of a house:

Rule: All people in that house are German
Fact: All these people came out of that house

Conclusion: All these people are German

In the following example of induction, a conclusion and a fact are used to arrive at a rule. The conclusion may be the result of an experiment or a deduction. The fact may be the outcome of an experiment and in this case the experiment of watching who comes out of a particular house.

Conclusion: All these people are German
Fact: All these people came out of that house

Rule: All people in that house are German

The rule arrived at by induction is true only in a probabilistic sense. As the number of outcomes due to new trials of the experiment increases, the probability assigned to the rule changes.

Induction is a very important mode of reason for arriving at rules we can later use with deduction to arrive at new conclusions, and so forth. Its is limited by our ability to conceive general rules to use in deductions that will provide new conclusions to be used in induction. It is also limited by our ability to make observations and record outcomes of experiments accurately enough to use in both deduction and induction.

The following is an example of abduction:

Conclusion: All these people are German
Rule: All people of that house are German

Fact: All these people came out of that house

Here we arrive at a fact, given a rule and a conclusion. The fact is actually a hypothesis or conjecture, and it is a best explanation but not in the same probabilistic sense of induction. Peirce formulated a few variations of abduction but the general idea remained the same. In essence, this is a mode of reason for making hypotheses about facts we cannot directly arrive at by experimentation and observation but are the possible best explanations or reasons.

Why is this last mode of reason so important? In mathematics, there are many facts, actually an infinite number of them. We can use some of these facts as rules in deductions with other facts, or with other rules to come up with new conclusions. Then, we can use other facts as rules and the new conclusions to get more abducted facts, and so on, ad infinitum. In physics, we can use facts in deductions or inductions to come up with new conclusions or rules. When all modes of reason are combined, reason, in the sense of facts, causes, proofs, and explanations has no limits. Below is an example from physics that illustrates how abducted facts can be used as rules:

The Michelson-Morley experiment performed in 1887 is probably one the most important scientific studies ever performed. The conclusion from the experimental data was that there is no luminiferous anther, meaning that there is not a light-bearing medium. Albert Einstein used this conclusion in conjunction with a set of rules already developed by other physicists, primarily by Maxwell and Lorentz, to abduct the hypothesis that the speed of light is constant in all globally inertial reference frames. This hypothesis was raised to the status of a principle in the Theory of Special relativity. This principle cannot be proved directly, in the sense of proving that the one-way speed of light is constant in all globally inertial reference frame, but in conjunction with other principles new predictions were possible that were confirmed by experiment. None of the predictions made by this theory has even been falsified by an experiment. This example shows how an abducted hypothesis about the motion of light that is not provable led to one of the most successful theories of physics. The important thing to realize is that Einstein had the choice of abducting a host of other similar hypotheses. One could attribute his final selection of a best explanation to a faculty that goes beyond the known modes of reason and has to do with creativity, intelligence and ingenuity.

The problem of abduction is the potential of generating a plurality of implications that involve a false abducted hypothesis. It is know that every false proposition implies a true proposition, a problem in logic known as the *paradox of material implication* [4]. For example, the two implications "If the moon is made of green cheese, then life exists on other planets" and "If life exists on other planets, then life exists on earth" are both true, regardless of whether life on other planets exists.

Implications like the above are called fallacies of relevance. In physics, it is a lot easier to spot fallacies of relevance. Most implications in Physics are entailments, meaning that the antecedent and the consequent exhibit high degree of relevancy. However, in Mathematics it may be hard to spot irrelevancies of this kind since mathematical objects are arrangements of symbols or bits.

It may be highly improbable to ever arrive at a theory of everything for all of mathematics, that has a final conclusion (set of theorems) and a final rule (axiom/principle set) in the face of an infinite number of facts. This is not due to any limits of reason but maybe due to our inability to determine which of the many facts are important to be used as rules and which ones are irrelevant to the theory. In physics, the difficulty for arriving at a theory of everything is mainly due to a limited ability to discover new facts experimentally. We thus resort to abduction, like for instance a hypothesis about the speed of light discussed above, the existence of dark energy or the Higgs boson and other facts that are just probable but not certain at all and very difficult to confirm experimentally with our measuring capabilities.

3. Discussion

Chaitin may be correct that there can never be a theory of everything for all of mathematics but also for physics. However, not because of any limits of reason, because there is none. When all known modes of reason are properly accounted for, the mathematical facts that Chatin claims that are true for no reason are actually part of reason. The reason for these mathematical facts is the reason used to arrive at them and this is the important point to understand. Their existence does not impose any limitation on reason since it was reason that produced them in the first place. However, there are possibly limits of our ability to decide which facts we must elevate to the status of a principle in order to arrive at a possible theory of everything. I argue that if reason, as a type or process of thought involves much more than facts, proofs, causes, or explanation, then such a theory may be possible. We may be missing other important modes of reason we do not know at this point what they are and how they operates and they may involve the relation of reason to reality, regardless of whether it is depended or independent of reason.

References

1. Chaitin, G. J.,"The Limits of Reason", Scientific American, March 2006 http://www.cs.umaine.edu/~chaitin/sciamer3.pdf

2. Isaac Newton's *Principia*, Translated by Andrew Motte, Prometheus, 1995.

3. Peirce, C. S. *Collected Papers of Charles Sanders Peirce*, edited by Charles Hartshorne, Paul Weiss and Arthur W. Burks, 8 vol., Cambridge (Massachusetts), Harvard University Press, 1931-1966.

4. Peter Suber, http://www.earlham.edu/~peters/courses/log/mat-imp.htm

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