How Norton and Roberts Tried to Fix a 350 Year-Old Blunder with a New Blunder

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Abstract

In this short paper I show the trivial mistake made in a paper by Norton and Roberts in an attempt to analyze and partially save an argument supposedly made by Galileo in refutation of the speed-distance law of fall.

1. Introduction

The following paragraph is attributed to Galileo as asserted in [1] (page 1):

> When speeds have the same ratio as the spaces passed or to be passed, those spaces come to be passed in equal times; if therefore the speeds with which the falling body passed the space of four braccia were the doubles of the speeds with which it passed the first two braccia, as one space is double the other space, then the times of those passages are equal; but for the same moveable to pass the four braccia and the two in the same time cannot take place except in instantaneous motion. But we see that the falling heavy body makes its motion in time, and passes the two braccia in less [time] than the four; therefore it is false that its speed increases as the space.

I highly doubt Galileo made such statement. I would think that given the magnitude of the man and his approach to physics, he probably made a statement like:

> When speeds have the same ratio as the spaces passed or to be passed, those spaces come to be passed in equal times; if therefore the speeds with which the falling body passed the space of four braccia were the doubles of the speeds with which it passed the first two braccia, as one space is double the other space, then the times of those passages are equal; (*up to this point it is the same statement*).
>
> and this is a result that fails to confirm with observation (*added by me*), therefore it is false that its speed increases as the space (*Same conclusion as before*).

Regardless of who made the original argument, he made a blunder. Norton and Roberts try to partially fix it with another blunder in [1], a division by zero, which is not a permissible mathematical operation.

1. Norton's and Robert's blunder

In [1] there is an elaborate and often trivial analysis leading to the following claim (page 18):

> "That the speed-distance law requires a body to fall for an infinite time from rest to achieve any finite distance is most easily seen with a little calculus. For simplicity, let us take the case of speed v=dx/dt numerically equal to distance x. Then the fall is governed by the differential equation dx/dt = x. This equation has the familiar solution
>
> $$\frac{x}{x_1} = \exp(t - t_1) \qquad (1)$$
>
> It tells us that, if a body has arrived as position x1 at time t1, then, if it continues to fall for an additional time t – t1, it will have arrived at position x. If we select x1=0 as our initial point, then the ratio x/x1 diverges. It follows immediately that the time t - t1 needed to fall to any finite position x>0 is infinite."

We know from elementary math that division by 0 is not a permissible operation. When one is allowed to divide by zero he can get all sorts of strange results, for example that 1 = 2. I wonder why Norton and Roberts are willing to divide by zero. A better approach is to re-arrange (1) as follows:

$$x = x_1 \exp(t - t_1) \qquad (2)$$

Now, if we select $x_1 = 0$, we get $x = 0$ from (2), for all t > 0 other than t = ∞ . This means that motion cannot take place if a body starts at $x = 0$ in any finite time interval. No matter how time evolves, whether universal or relative, the body stays at the same initial position. The motion becomes undefined only at t = ∞ because at that point we have the operation x = 0 × ∞ the result of which is undefined. What does this mean, one might ask?

My answer is that I do not know and I am not a position to know. A universe where free-fall is governed by the speed-distance law (simply stated a law of the form $v = kx$, where k is a positive constant) is an impossible one unless some interventions and interactions take place, the essence of which is beyond the scope of this paper. For example, motion could be in that case grounded on *Occasionalism*, an interaction mentioned in [2] (section 6).

What is important to realize is that Norton and Roberts made a trivial blunder and via a division by zero reached an erroneous conclusion. The reason that the argument attributed to Galileo is a blunder is the fact that, with a speed-distance law, motion cannot even begin, as proper application of Algebra shows. Think about it: why should motion begin if speed is proportional to distance? In the free fall condition of classical mechanics bodies move because their speed changes as a function of time according to dv/dt = g(x). This means that nature affects a change in the velocity of the body so that its time rate of change is equal to the acceleration of gravity g(x), which varies with the square of the distance of fall and does not depend on the mass of the falling body.

However, in a free-fall governed by a speed-distance law, how could nature affect a velocity change? This could be a simple explanation offered in the form of a question. Often, it turns out that the best answer is a question. There may be other more complicated explanations that are beyond the scope of this paper. But I think this simple analysis suffices to show two things:

(1) The original argument attributed to Galileo is a blunder

(2) The analysis in [1] is based on the trivial blunder of division by zero.

Conclusion

Infinite time of fall cannot even remotely save the argument attributed to Galileo about the speed-distance law applied to free-fall because it is based on the trivial error of dividing by zero. It is clear that the argument attributed to Galileo is false because proper Algebra shows motion cannot even begin with a speed-distance law. In such law, time and motion are not inextricably related, contrary to Newtonian Mechanics, and are independent variables with no obvious physical connection.

References

[1] Norton, John D. and Roberts, Bryan W. (2010) Galileo's Refutation of the Speed-Distance Law of Fall Rehabilitated . http://philsci-archive.pitt.edu/archive/00005479/

[2] E. Harokopos (2005) Power as the Cause of Motion and a New Foundation of Classical Mechanics, Progress in Physics, v2 http://www.digitalcosmology.com/Articles/articles.html

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