

The Dyson Number Problem

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Problem Definition:

We are looking for integer numbers N_D , called the *Dyson numbers* [1], such that

$$\alpha N_D = D[N_D] \quad (1)$$

where D is defined as the *Dyson operator* that takes the last digit of a number N_D with n digits and places that digit to the front of the number, while α is an integer number between 2 and 9.

Answer:

Let N_D be a Dyson number of n digits. Then

$$N_D = \sum_{k=1}^n x_k 10^{k-1} \quad (2)$$

where x_k is the k th digit

The Dyson operator D takes the last digit x_1 and places it in the front of the number as follows:

$$D[N_D] = x_1 10^{n-1} + \sum_{k=2}^n x_k 10^{k-2} \quad (3)$$

According to (1) – (3), we are looking for n and α , such that,

$$\alpha \sum_{k=1}^n x_k 10^{k-1} = x_1 10^{n-1} + \sum_{k=2}^n x_k 10^{k-2} \quad (4)$$

Multiply each side of (4) by the number 10

$$10\alpha \sum_{k=1}^n x_k 10^{k-1} = x_1 10^n + \sum_{k=2}^n x_k 10^{k-1} \quad (5)$$

Now add and subtract x_1 from the right hand side of (5). The result is:

$$10\alpha \sum_{k=1}^n x_k 10^{k-1} = x_1 10^n + \sum_{k=1}^n x_k 10^{k-1} - x_1 \quad (6)$$

With the help of (2), (6) transforms as follows:

$$10\alpha N_D = x_1 10^n + N_D - x_1 \quad (7)$$

Solving for the Dyson number we get

$$N_D = x_1 \frac{10^n - 1}{10\alpha - 1} \quad (8)$$

We can see immediately that if N_D must have n digits in (8), then x_1 , the first digit, cannot be equal to 1.

(8) is the general equation for finding the Dyson numbers. Existence of such numbers requires that the value of the ratio r below times x_1 is an integer number:

$$r = \frac{10^n - 1}{10\alpha - 1}$$

In the case that $\alpha = 2$ and $n = 18$, we can compute $r = 52631578947368421$

Thus, for $x_1 = 2$, $N_D = 105263157894736842$

The Dyson operator gives the number: 210526315789473684

We can see that $2 \times 105263157894736842 = 210526315789473684$

By exhaustive search all Dyson numbers can be found for $x_1 > 1$, $2 < \alpha < 9$ and $n > 1$

This is not a trivial problem since the Dyson numbers are function of x_1 , n and α , and r times x_1 must be an integer number. This is a highly non-linear function with 3 unknowns and one non-linear constraint that is a function of all unknowns.

References

1. <http://tierneylab.blogs.nytimes.com/2009/04/06/freeman-dysons-4th-grade-math-puzzle/>
2. <http://www.digitalcosmology.com/Articles/articles.html#dyson>